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OPTIMAL EVASIVE MANEUVERS IN CONDITIONS OF UNCERTAINTY*

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0135

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Optimal missile avoidance by an evader not having information on the relative state is analysed. This problem is equivalent to the homing of a missile of known parameters against a randomly maneuvering target. The objective is to maximize the R.M.S. miss distance by selecting the optimal frequency of a periodical maneuver with random initial phase. Comparison to results of missile avoidance with perfect information shows that the maximized R.M.S. miss distance can reach 60-80% of the optimal deterministic value.

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I. INTRODUCTION

The problem of an airplane evading a homing missile has been formulated in the past as a deterministic optimal control problem.[1,2,3,4] Such formulation implicitly assumes the existence of perfect information on the relative state (missile position and closing velocity), and missile's parameters. Practically, perfect information implies that the pilot is allerted that a missile of known type has been launched against his airplane; the relative state can be measured and the time of interception can be estimated.

However, in a real environment this condition is not satisfied. There are several source of information imperfections:

a) Lack of intelligence or identification (parameter uncertainty);

b) Unaccessible state variables (partial observability);

c) Unaccurate state variables (measurement error or noise);

d) Non existent threat warning (no initial conditions).

Consequently, the real missile avoidance problem is of a stochastic nature. The mathematical formulation of such a problem is strongly dependent on the available information.

In case of parameter uncertainty or noisy measurements it is possible to formulate an optimal stochastic control problem, with the R.M.S. miss distance as a pay-off.[5,6] In this paper it is assumed that the evading airplane has no information on the relative state (including initial conditions), but missile parameters are known. Analysis of this problem is guided by the results obtained for deterministic missile avoidance:[1-4] a "bang-bang" type (limit load factor) optimal maneuver with a switch function which depends on the parameters of the problem and the relative state.

Since in the present problem the relative state is unknown the optimal maneuver must be random. Three types of random maneuvers are of interest:

a. Maneuver of limit load factor with random switching time of Poissonian distribution, called the "Random Telegraph Maneuver" (R.T.M.).

b. Periodical maneuver with equally distributed random starting time.

c. Periodical maneuver with equally distributed random phase.

The optimal missile avoidance problem can be transformed for each case to the problem of a missile homing against a randomly maneuvering target.

The R.M.S. miss distance of a proportionally homing missile against a target with a random phase sinusoidal maneuver was calculated in the past.[7,8] "Random Telegraph Maneuver" was also studied.[6,8] A recent work[8] compared R.T.M. and random periodical maneuvers and indicated that for long flight times:

(i) A periodical, square wave type maneuver guarantees larger RMS miss distances than the comparable RTM.

(ii) Periodical maneuvers of random initial phase generate the same miss distances than a randomly starting maneuver of the same frequency.

Previous works used to analyse homing performance from the missile's point of view, considering random target maneuvers as some given stochastic input. In the present paper, motivated to enhance aircraft survivability in situations where relative state information is not available, the emphasis is to investigate the characteristics of random maneuvers which maximize the R.M.S. miss distance. No similar analysis is known in the existing technical literature. Based on the results of the above mentioned recent study,[8] the present analysis concentrates on periodical target maneuvers with random initial phase and the objective is to determine the optimal frequency and the resulting R.M.S. miss distance as functions of missile and target parameters.

In Section II the assumptions and the mathematical formulation are presented. In the sequel the miss distance due to target maneuver is expressed in a closed form in the complex domain, using adjoint analysis.[7,10-11] In Section III the closed form solution is extended to compute R.M.S. miss distances generated by random stationary target maneuvers and implemented for a periodical maneuver of random initial phase. In Section IV the optimal maneuver frequency and the resulting R.M.S. miss distance are computed for unlimited missile acceleration. In Section V effects of bounded missile acceleration on the optimal frequency and the miss distance is shown.

II. MATHEMATICAL FORMULATION

The analysis is based on the following set of assumptions:

1) The missile is guided by proportional navigation using a constant effective navigation ratio N'.
2) Aircraft trajectory during the evasive maneuver does deviate much from its initial direction.
3) Missile and target are considered as constant speed point-mass elements.
4) Missiles are launched in a collision course.
5) Both missile and target perform lateral accelerations perpendicular to the initial line of sight.
6) The deviation of the trajectory form the reference line of sight can be decomposed in two perpendicular planes. For the sake of simplicity only one of these planes is considered and the gravity component in this plane is neglected.
7) Missile dynamic is taken into account (first order transfer function with a time constant τ) but its acceleration is unbounded.
8) Target lateral acceleration and roll rate are bounded.
9) The missile flight time t_f is long compared to its equivalent first order time constant.

It is believed, based on the experience of similar deterministic studies,[2,4] that these assumptions provide the necessary insight for a realistic analysis.

The geometry of the interception for missile launched in a collision course (Ass. 4) is defined in Fig. 1. The equations of the linearized trajectory, based on assumptions 1-6 were developed in previous papers[2,4] and are not repeated here. This mathematical model is summarized in the block diagram in Fig. 2, where $F(s)$ is the transfer function of the guidance loop. In the linearized kinematic model the miss distance is defined by

$$m_f \triangleq y(t_f) = \lim_{t \to t_f} (y_T - y_M) \qquad (1)$$

It can be calculated by

$$m_f = \int_0^{t_f} g(t_f, t) y_T(t) dt \qquad (2)$$

where $g(t_f,t)$ is the impulse response of the closed loop linear time-varying dynamic system shown in Fig. 2. Equation (2) generally does not yield a closed form solution in the time domain. It can be, however, solved analytically in the complex (Laplace) domain using adjoint analysis.[10,11]

The adjoint of any time-varying linear system can be easily constructed from the block diagram of the original system observing the following rules:

1) Reverse all signal flow.
2) Redefine branch points as summation junctions and vice versa.
3) All time-invariant elements remain unchanged.
4) Replace t by $(t_f - t^a)$ in the argument of all time-varying elements.

5) The input to the adjoint system is a unit impulse $\delta(t^a)$, whose application point corresponds to the output of interest in the original system.
6) The output of each element in the adjoint system represents the output of interest in the original system, due to a unit impulse disturbance at the input of the same element.

In the present problem the output of interest is the miss distance due to target maneuver. In Fig. 3 the block diagram of this adjoint system is shown.

The impulse response functions of the original and the adjoint systems respect the following relationship[12]

$$g(t_f,t) = g^a(t^a,0) \tag{3}$$

with

$$t^a = t_f - t \tag{4}$$

Consequently, Eq.(2) can be written as

$$m_f = \int_0^{t_f} g^a(t^a,0)y_T(t_f-t^a)dt^a \tag{5}$$

This expression, being a convolution type integral, yields the following Laplace transform with respect to the adjoint time variable t^a

$$m_f(s) = g^a(s)y_T(s) \tag{6}$$

From Fig. 3 it is easy to verify that $g^a(s)$ satisfies the differential equation

$$\frac{dg^a(s)}{ds} = N' \frac{F(s)}{s} g^a(s) \tag{7}$$

The solution of Eq.(7) is

$$g^a(s) = C \exp \left\{ N' \int^{s} \frac{F(\xi)}{\xi} d\xi \right\} \tag{8}$$

The constant of integration can be obtained from the Initial Value Theorem

$$\lim_{t^a \to 0} m_f = \lim_{s \to \infty} s\, m_f(s) = \lim_{s \to \infty} s, g^a(s) \quad y_T(s) \tag{9}$$

The miss distance, generated by a unit step target motion at $t^a = 0$, i.e.:

$$y_T(s) = \frac{1}{s} \tag{10}$$

is equal to a unit displacement. Thus Eq.(9) leads to

$$\lim_{s \to \infty} g^a(s) = 1 \tag{11}$$

and consequently

$$g^a(s) = \exp \left\{ N' \int_{\infty}^{s} \frac{F(\xi)}{\xi} d\xi \right\} \tag{12}$$

Once $g^a(s)$ is known, the miss distance due to target maneuver is obtained from Eq.(6) by

$$m_f = L^{-1}\{g^a(s)y^T(s)\} \tag{13}$$

For integer values of N' and transfer functions of simple form $g^a(s)$ can be expressed in a closed form (see Table 1)

TABLE 1. The function $g^a(s)$ for 3 types of transfer functions.

F(s)	$g^a(s)$
$\dfrac{1}{1+\tau s}$	$\left[\dfrac{\tau s}{1+\tau s}\right]^{N'}$
$\dfrac{1}{\left(1+\frac{\tau}{2}s\right)^2}$	$\left[\dfrac{\tau s}{2+\tau s}\right]^{N'} \exp\left[\dfrac{2N'}{2+\tau s}\right]$
$\dfrac{1}{(1+\tau_1 s)(1+\tau_2 s)}$ $\tau_1+\tau_2=\tau$ $\quad 0 \leqslant a \leqslant 1$ $\tau_1 = a\tau_2$	$\left[(a)^{\frac{1}{1-a}} \cdot \dfrac{\tau s a}{1+a} \cdot \dfrac{\left(1+\frac{a\tau s}{1+a}\right)^{\frac{a}{1-a}}}{\left(1+\frac{\tau s}{1+a}\right)^{\frac{a}{1-a}}}\right]^{N'}$

III. R.M.S. MISS DISTANCE DUE TO RANDOM STATIONARY TARGET MOTION

The concept of adjoint system is very effective to analyse behavior of linear systems with stationary stochastic inputs.[7,11] The square of the average miss distance due to some random stationary target motion is given for long times of flight[7] by

$$\lim_{t_f \to \infty} \overline{m}_f^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi_{y_T}(\omega) \; |g^a(j\omega)|^2 \, d\omega \tag{14}$$

where $\psi_{y_T}(\omega)$ is power spectral density of the target motion.

For a periodical target motion with random initial phase

$$y_T(t) = y_{T_0} \sum_{n=1}^{\infty} b_n \sin(n\omega_T t + \phi) \tag{15}$$

the power spectral density is given by

$$\psi_{y_T}(\omega) = y_{T_0}^2 \frac{\pi}{2} \sum_{n=1}^{\infty} b_n \left[\delta(\omega - n\omega_T) + \delta(\omega + n\omega_T) \right] \tag{16}$$

where δ is the Dirac impulse function. Substitution into Eq.(14) yields

$$\lim_{t_f \to \infty} \overline{m}_f^2 = \frac{y_{T_0}^2}{2} \sum_{n=1}^{\infty} b_n^2 \, |g^a(nj\omega_T)|^2 \tag{17}$$

Though theoretically t_f has to be infinitely large, the right side of Eq.(17) is valid for values of t_f exceeding 5-6 guidance time constants. For shorter flight times the ensemble average of the miss distances does not reach a constant asymptotic value. This short duration problems are investigated in a separate report.[13]

For illustration purposes let us apply Eq.(17) for a pure sinusoidal target maneuver of equally distributed random initial phase

$$\ddot{y}_T(t) = a_T \sin(\omega_T t + \phi) \tag{18}$$

$$P_\phi(\beta) = \begin{cases} \frac{1}{2\pi} & -\pi \leq \beta \leq \pi \\ 0 & \text{elsewhere} \end{cases} \tag{19}$$

and first order missile dynamics. From Table 1 we find

$$g^a(s) = \left(\frac{\tau s}{1 + \tau s}\right)^{N'} \tag{20}$$

double integration of (18) results in

$$y_T(t) = -\frac{a_T}{\omega_T^2} \sin(\omega t + \phi) \tag{21}$$

and substitution of (20) and (21) into Eq.(17) leads to

$$\lim_{t_f \to \infty} \left(m_f^2\right)_{\sin} = \frac{a_T^2}{2\omega_T^4} \left(\frac{\omega_T^2 \tau^2}{1 + \omega_T^2 \tau^2}\right)^{N'} \tag{22}$$

Introducing the normalized frequency

$$u \triangleq \omega_T \tau \tag{23}$$

and defining the normalized R.M.S. miss distance for long flight times $\overline{M}$ by

$$\overline{M}^2 \triangleq \lim_{t_f \to \infty} \frac{\overline{m}_f^2}{a_T^2 \tau^4} \tag{24}$$

Eq.(22) is transformed to

$$\overline{M}^2_{sin} = \frac{1}{2} \frac{u^{2N'-4}}{(1+u^2)^{N'}} \tag{25}$$

For a generalized periodical target maneuver of random phase

$$\ddot{y}_T(t) = a_T \sum_{n=1}^{\infty} C_n \sin(n\omega_T + \phi) \tag{26}$$

the coefficients b_n in Eq.(15) are determined by

$$b_n = \frac{C_n}{n^2 \omega_T^2} \tag{27}$$

For such maneuver $\overline{M}$ is given by

$$\overline{M} = \frac{1}{\sqrt{2}} \left[\sum_{n=1}^{\infty} C_n^2 \frac{(nu)^{2N'-4}}{(1+n^2u^2)^{N'}} \right]^{1/2} \tag{28}$$

IV. MANEUVER OPTIMIZATION

The normalized R.M.S. miss distance for long time of flight $\bar{M}$ depends on missile guidance parameters (as $F(s)$, N'), on the type of the periodical target maneuver (characterized by the Fourier coefficients C_n, $n=1,\ldots,\infty$) and the normalized frequency u. For any given missile and target maneuver type *there exists an optimal frequency* maximizing $\bar{M}$.

Three types of random phase periodical target maneuvers, shown in Fig. 4 are considered:

(i) Sinusoidal maneuver as given in Eq.(18).

(ii) Square wave type maneuver, defined by

$$\ddot{y}_T(t) = a_T \text{ sign } \{\sin(\omega_T t + \phi)\} \tag{29}$$

This maneuver assumes that target acceleration can be reversed instantaneously implying infinite roll rate. Aircraft roll-rate limitation can be considered by:

(iii) Truncated sine maneuver, defined as

$$\ddot{y}_T(t) = a_T \text{ sat } \left\{\frac{A_T \sin(\omega_T t + \phi)}{a_T}\right\} \tag{30}$$

where

$$A_T = a_T/\sin(\omega_T t_r) \tag{31}$$

t_r being the time required to roll the airplane 90 degrees at maximum roll rate. The Fourier coefficients of the truncated sine maneuver are

$$C_1 = \frac{4}{\pi}\frac{1}{\sin(\omega_T t_r)}\left[\frac{\omega_T t_r}{2} + \frac{1}{4}\sin(2\omega_T t_r)\right]$$

$$C_n = \frac{4}{\pi}\frac{\cos(n\omega_T t_r)}{n} \quad ; \quad n = 2k+1 \tag{32}$$

$$C_n = 0 \qquad n = 2k \qquad k = 1,2,\ldots$$

The square wave maneuver can be considered as the limit case with $t_R = 0$, yielding for all the odd coefficients

$$\left(C_n\right)_{sq} = \frac{4}{\pi}\ \frac{1}{n} \tag{33}$$

The other limit case is the pure sinusoidal maneuver with $t_r = \pi/2\omega_T$ for which $\left(C_1\right)_{sin} = 1$.

Substituting the expression of normalized frequency u from Eq.(23) into Eq.(32) yields

$$C_1 = \frac{2}{\pi}\ \frac{1}{\sin(4\theta_r)}\left[4\theta_r + \frac{1}{2}\sin(2u\theta_r)\right]$$

$$C_n = \frac{4}{\pi}\ \frac{\cos(nu\theta_r)}{n} \quad ; \quad n = 2k+1 \tag{34}$$

with

$$\theta_r = \frac{t_r}{\tau} \tag{35}$$

The optimal normalized frequency u^*, maximizing $\overline{M}$, can be obtained from

$$\frac{d\overline{M}^2}{du} = 0 \tag{36}$$

or by substitution of (17), (23), (24) and (27)

$$\frac{d}{du}\left(\sum_{n=1}^{\infty} C_n^2 \frac{1}{n^4 u^4} |g^a(jnu)|^2\right) = 0 \tag{37}$$

It can be shown that the infinite series in Eq.(37) is convergent, each of its terms is differentiable and the series of the derivatives is uniformly convergent. Consequently differentiation can be carried out on each term separately, yielding to

$$\sum_{n=1}^{\infty} C_n^2 \frac{d}{du}\left(\frac{|g^a(jnu)|^2}{n^4 u^4}\right) = 0 \tag{38}$$

Due to the uniform convergence, the infinite series can be truncated to a finite sum with a prescribed negligible error

$$\sum_{n=1}^{N} C_n^2 \frac{d}{du}\left(\frac{|g^a(jnu)|^2}{n^4 u^4}\right) \leqslant \varepsilon \tag{39}$$

For any finite number of terms an algebraic equation is obtained, yielding a sequence of solutions u_N. This sequence converges, for $N \to \infty$, to the optimal value of normalized frequency u^*.

First Order Dynamics

In the example of a pure sinusoidal manner only the first term exists. Differentiation of Eq.(25) leads to the following expression for u*

$$u^{*2}_{sin} = \frac{N'-2}{2} \tag{40}$$

Substituting (40) into (25) gives the optimal normalized R.M.S. miss distance for long flight times

$$\bar{M}_{sin} = \sqrt{2}\left[\frac{(N'-2)^{N'-2}}{N'^{N'}}\right]^{1/2} \tag{41}$$

Detailed numerical analysis of the other type of maneuvers (square move and truncated sine) shown in Figs. 5 and 6 revealed that:

a) The optimal frequency is practically the same (with 99% of accuracy) for all periodical maneuvers as given by Eq.(40).

b) The ratio of the optimal RMS miss distances for different types of periodical maneuvers is equal to the ratio of their first Fourier coefficient.

These facts can be explained by the "bound pass" property of the equivalent transfer function $g^a(s)/s^2$ which alternates all higher frequencies. The ratio of the third harmonic to the first one in Eq.(28) is rather small

$$\frac{M_3}{M_1} = \frac{C_3}{C_1} \; 3^{N'-2}\left[\frac{1+4^2}{1+9^2}\right]^{N'/2} \ll 1 \tag{42}$$

as shown in Fig. 7.

As a consequence of this observation the normalized RMS miss distance can be computed by the simplified formula

$$\overline{M}^* \cong \sqrt{2}\, C_1 \left[\frac{(N'-2)^{N'-2}}{N'^{N'}}\right]^{1/2} \tag{43}$$

Since in the truncated sine maneuver C_1 is the function of θ_r (see Eq.(32)), the dependence of $\overline{M}^*$ on this parameter can be directly investigated as shown in Fig. 6.

Second Order Dynamics

The optimal maneuver frequency was calculated also for missiles of non oscillatory, second order transfer functions

$$F_a(s) = \frac{1}{(1+\tau_1 s)(1+\tau_2 s)} \tag{44}$$

where

$$\tau_1 = a\tau_2 \qquad < a \leq 1 \tag{45}$$

and

$$\tau_1 + \tau_2 = (1+a)\tau_2 = \tau \tag{46}$$

Results are plotted in Figs. 8 and 9 including a comparison to first order dynamics. These results reveal that for $N' > 4$ the optimal frequency against a second order missile should be lower than predicted by the first order approximation. For $2 \leq N' < 4$ there is no significant difference in the optimal maneuver frequency. The dependence of the miss distance on the pole ratio "a" is shown in Fig. 9 for $N' = 3$, indicating an increase of about 3 times between a double pole (a=1) and first order dynamics.

V. EFFECT OF LIMITED MISSILE MANEUVERABILITY

In the previous sections unbounded missile acceleration was assumed allowing the use of a linear mathematical model. Whenever the real limitations of missile maneuverability is taken into account the system becomes non-linear and as a consequence the adjoint analysis is no more valid.

Investigation of non-linear systems with random inputs is generally performed using "Monte Carlo" methods (an average of very large number of independent simulations). Since the "brute force" method is rather expensive and time consuming, it is used only if no more efficient methods apply.

Recently the method of covariance was extended to non-linear dynamic system. This new technique called CADET (Covariance Analysis, Describing function Technique) is described in Ref.14. The main advantage of CADET is that the stochastic properties of the non-linear system are obtained in a single computer run. However, in the computational process a relatively large number of differential equations have to be solved simultaneously. For a system of order "k", and random periodical maneuver, approximated only by the two first terms, the number of equations is

$$K_{CADET} = \frac{(k+5)(k+4)}{2} \tag{47}$$

For periodical random phase maneuvers the use of CADET may not be necessary. It was found[15] that due to the periodical nature of the miss distance a very precise ensemble average can be obtained from a small number (3-5) of simulations. Consequently, the number of differential equations to be solved are at most

$$K_0 = 5k < K_{CADET} \qquad \forall\ k \tag{48}$$

The results of 5 equally distributed initial phase simulations were compared[15] to a Monte Carlo analysis of 200 simulations, showing 99% of accuracy.

Results of the investigation $(u^*, \bar{M}^*)$ are plotted in Fig. 10 as the function of the target-missile maneuver ratio defined as

$$\eta \triangleq \frac{(\ddot{y}_T)_{max}}{(\ddot{y}_M)_{max}} = \frac{a_T}{a_M} \tag{49}$$

These results clearly indicate that the effect of limited missile maneuverability is to lower the optimal frequency and to increase the miss distance. This effect is due to the deterioration of the effective navigation gain during missile saturation. We can define

$$N'' = G_{equ} N' \tag{50}$$

$G_{equ} < 1$ being the equivalent amplification factor of the non-linear element. As shown in the previous section, lower value of N' results in increased miss distance and lower optimal maneuver frequency.

Finally, the results of the stochastic optimization were compared to the solution of optimal missile avoidance with perfect information.[2] The comparison, shown in Fig. 11, reveals that for the example chosen (N' = 4) the R.M.S. miss distance can reach 60-80% of the optimal deterministic value.

VI. CONCLUDING REMARKS

In this paper, optimal missile avoidance of an airplane which has no information on the relative state was investigated. Assuming the "worst case" of infinite missile maneuverability a linear time varying stochastic optimization problem was formulated and solved in a closed form using adjoint analysis. The solution provided the optimal maneuver frequency and the RMS miss distance for long missile flight times as functions of missile and aircraft parameters. Taking into account missile lateral acceleration limits resulted in a non-linear problem requiring numerical analysis.

Based on the periodical properties of the problem, a very efficient computational scheme could be used, yielding accurate results with a minimal computational effort. It was shown that in the case of limited missile maneuverability, lower optimal maneuver frequency is required and larger RMS miss distances are obtained.

Results of the stochastical optimization were compared to the optimal miss distances obtained in deterministic studies. This comparison indicated that the degradation of missile avoidance capability due to imperfect information may not be as serious as it could have been estimated. If the avoidance efficiency could be partially retained in the stochastic sense even for a total absence of state information, there is a definite hope that better (and probably satisfactory) performance could be achieved if partial or noise corrupted state measurements do exist.

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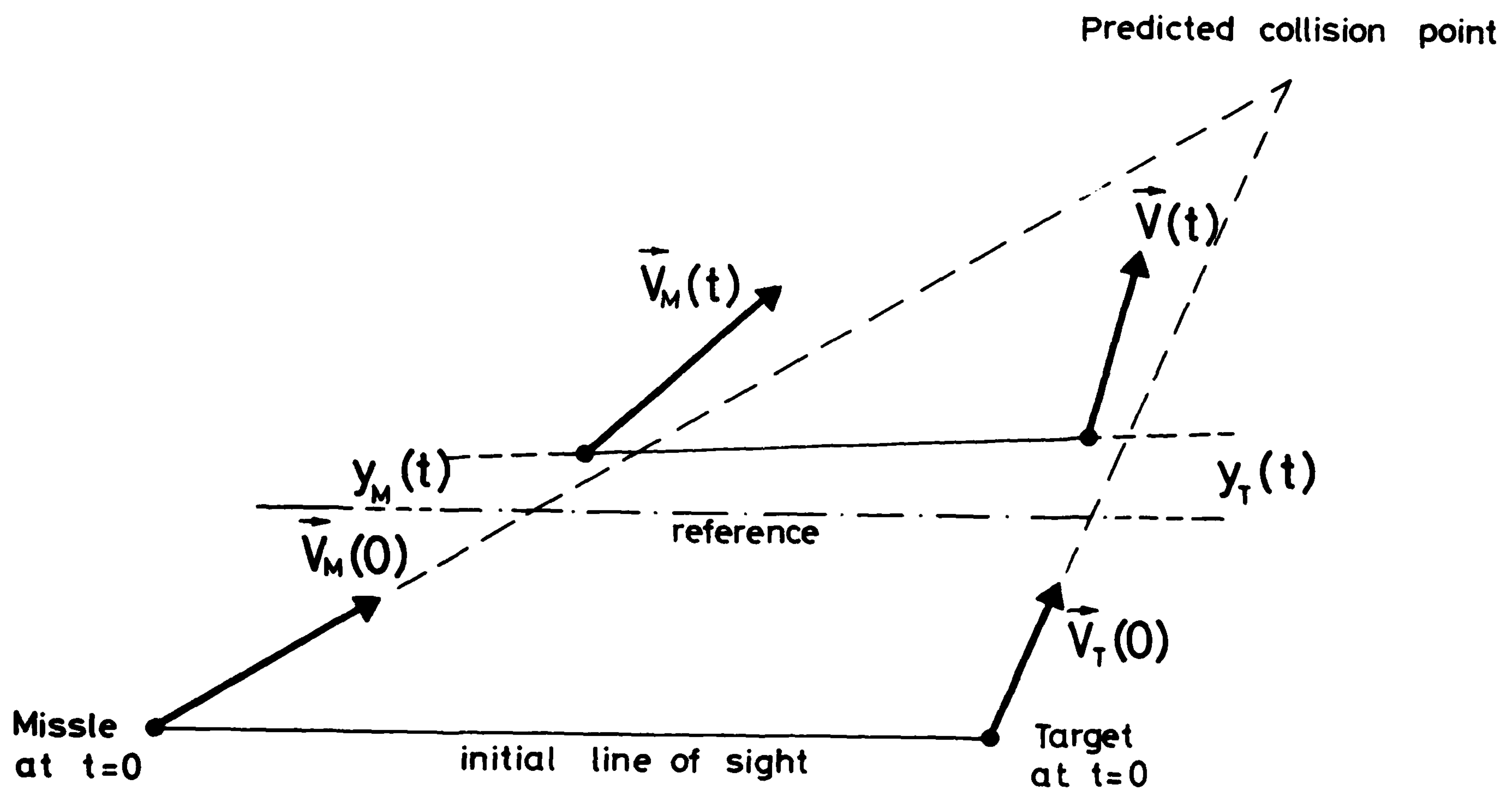

FIG. 1. Interception geometry.

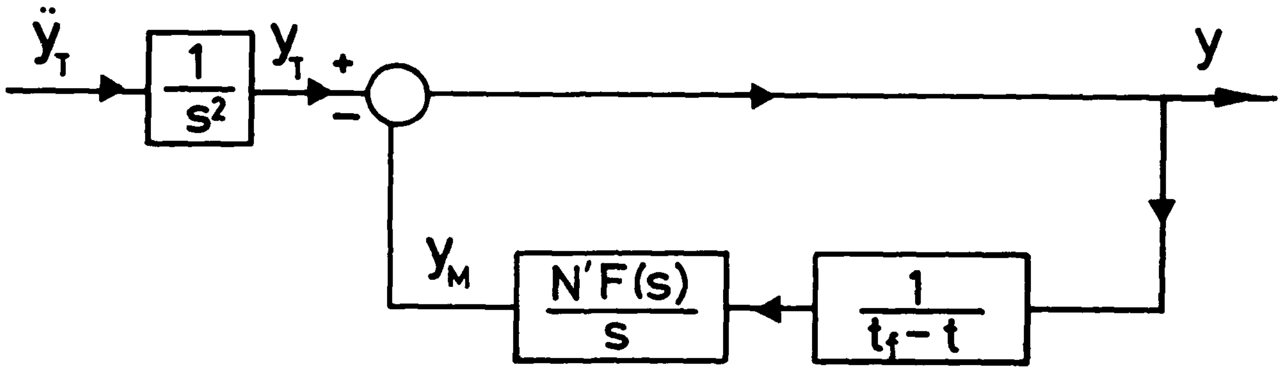

FIG. 2. Block diagram of 2-D proportional homing.

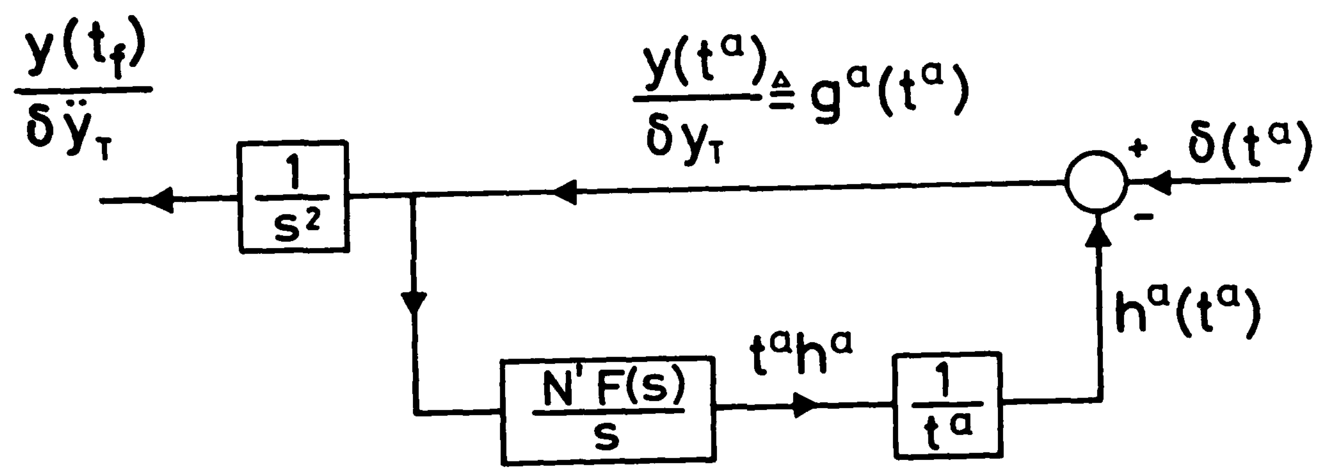

FIG. 3. Adjoint block diagram.

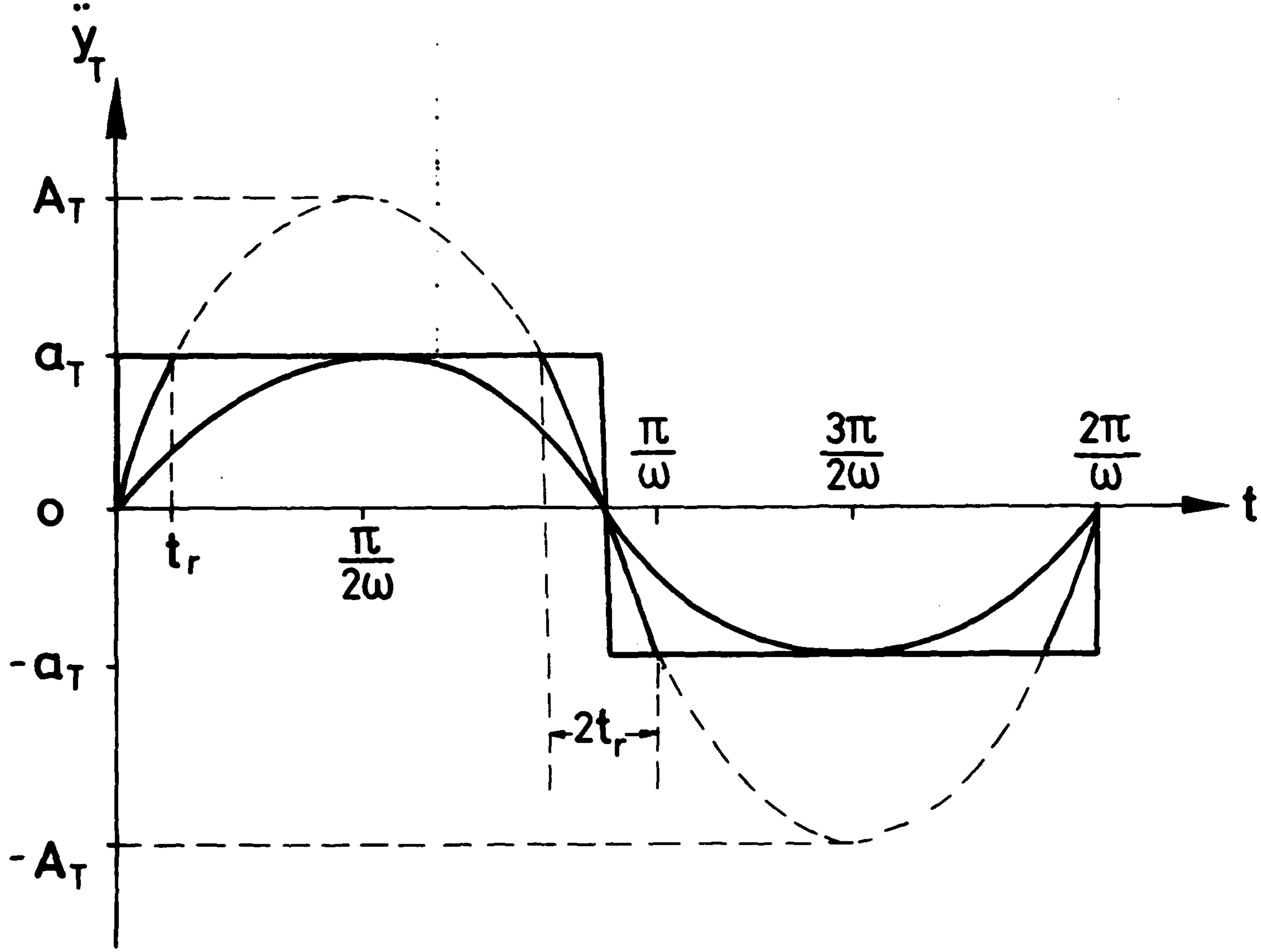

FIG. 4. Types of periodical maneuvers.

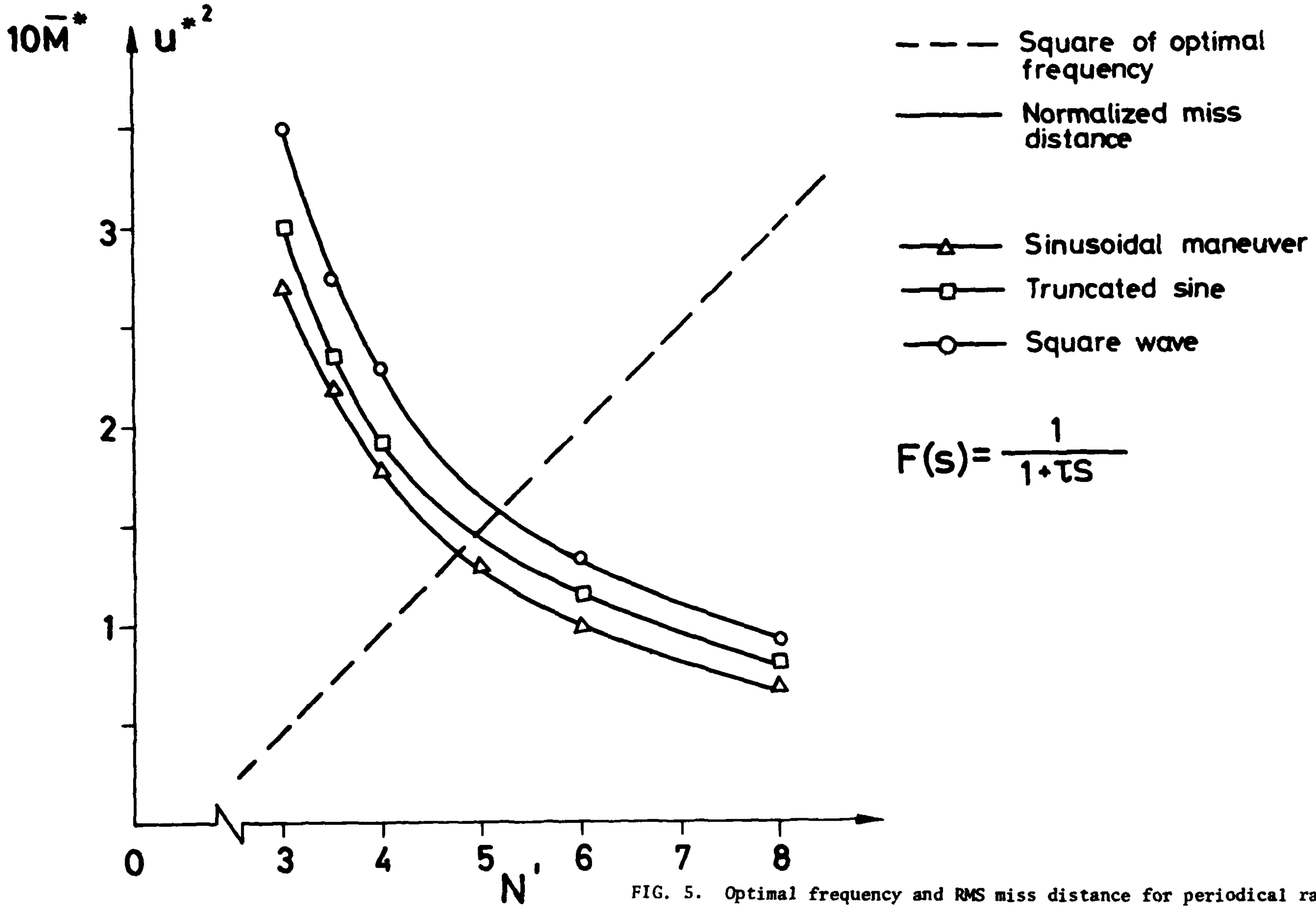

FIG. 5. Optimal frequency and RMS miss distance for periodical random phase maneuvers.

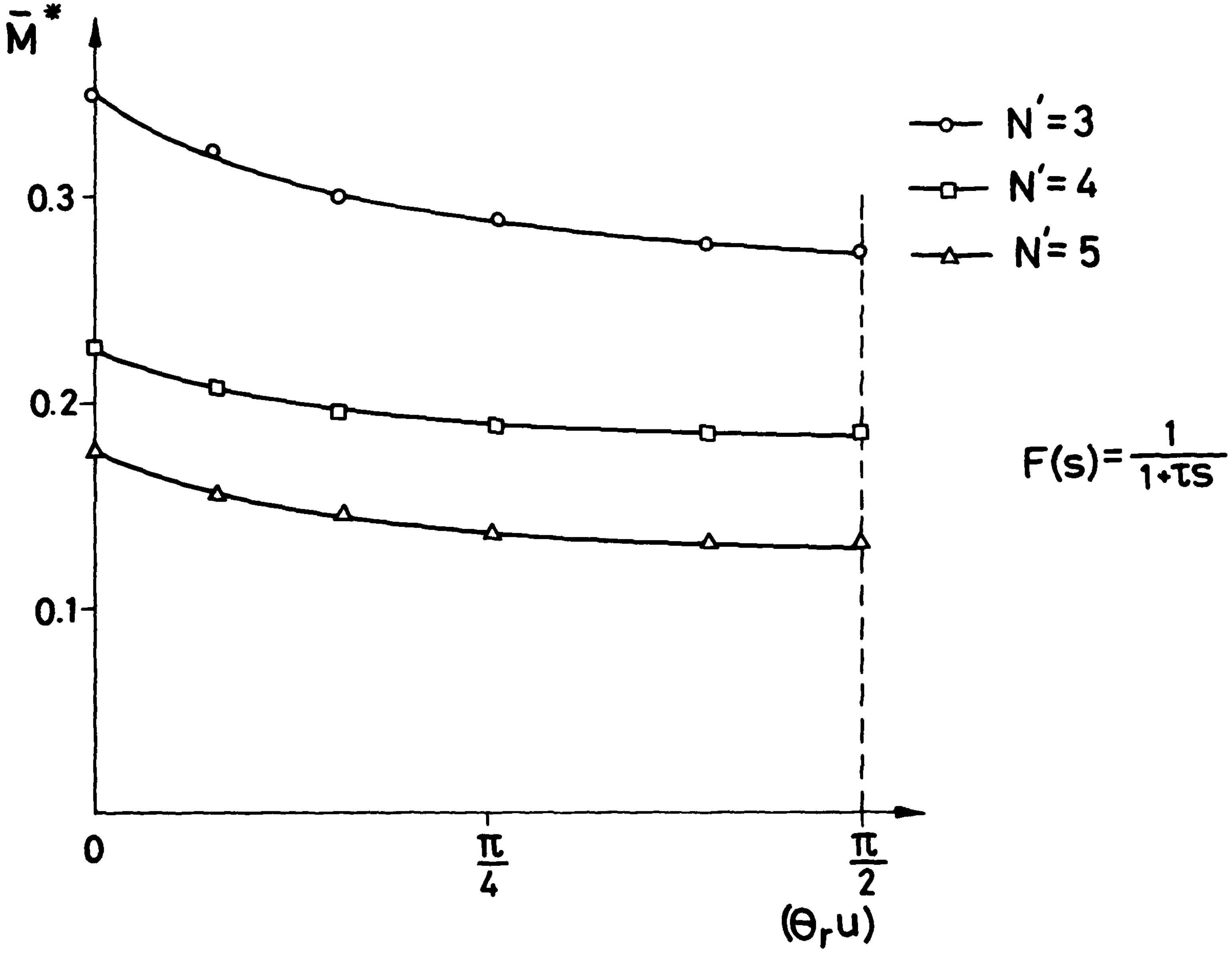

FIG. 6. Effect of target roll rate on the optimal RMS miss distance.

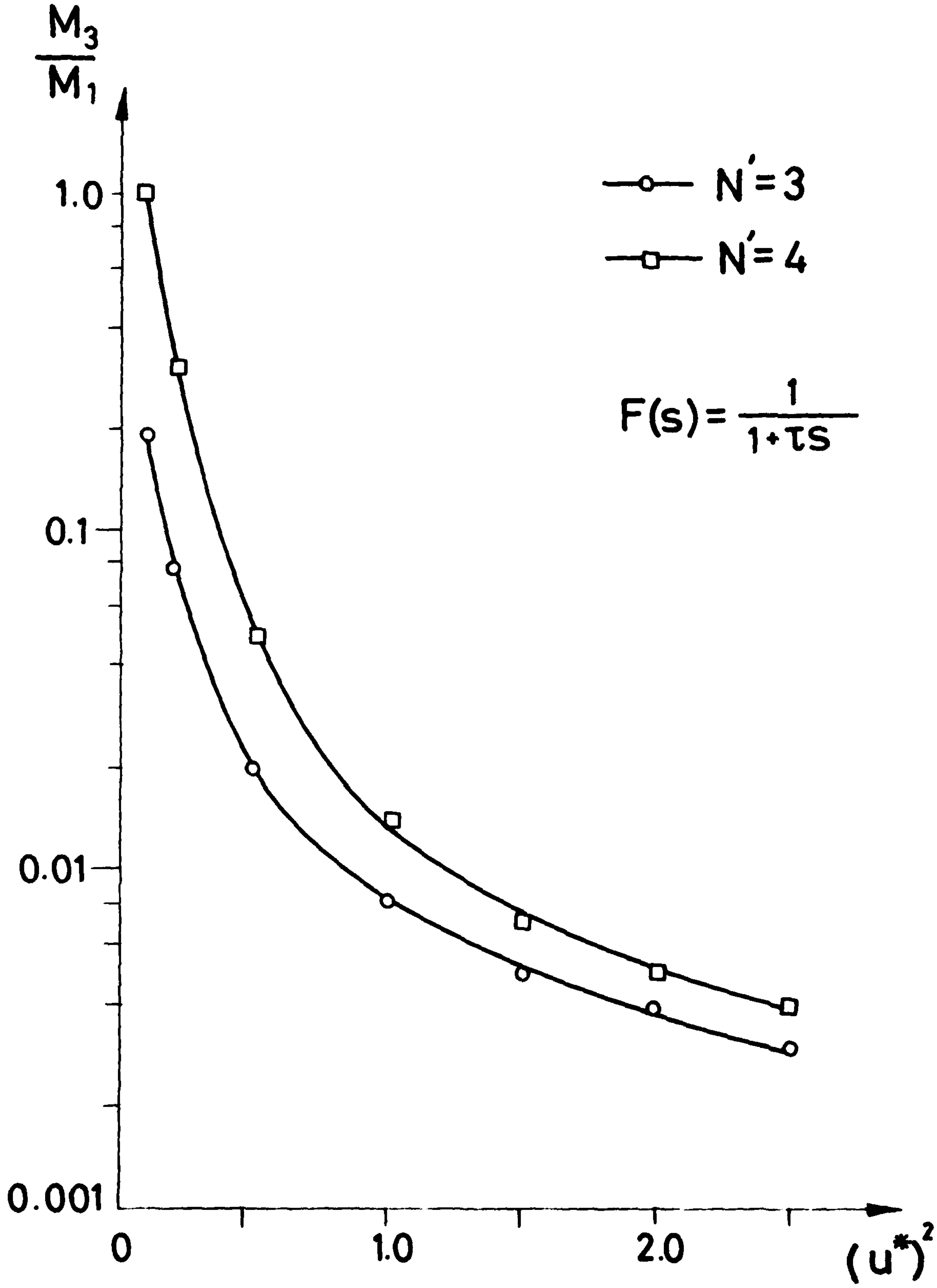

FIG. 7. The ratio of third and first harmonics in the formula of the miss distance due to periodical target maneuver.

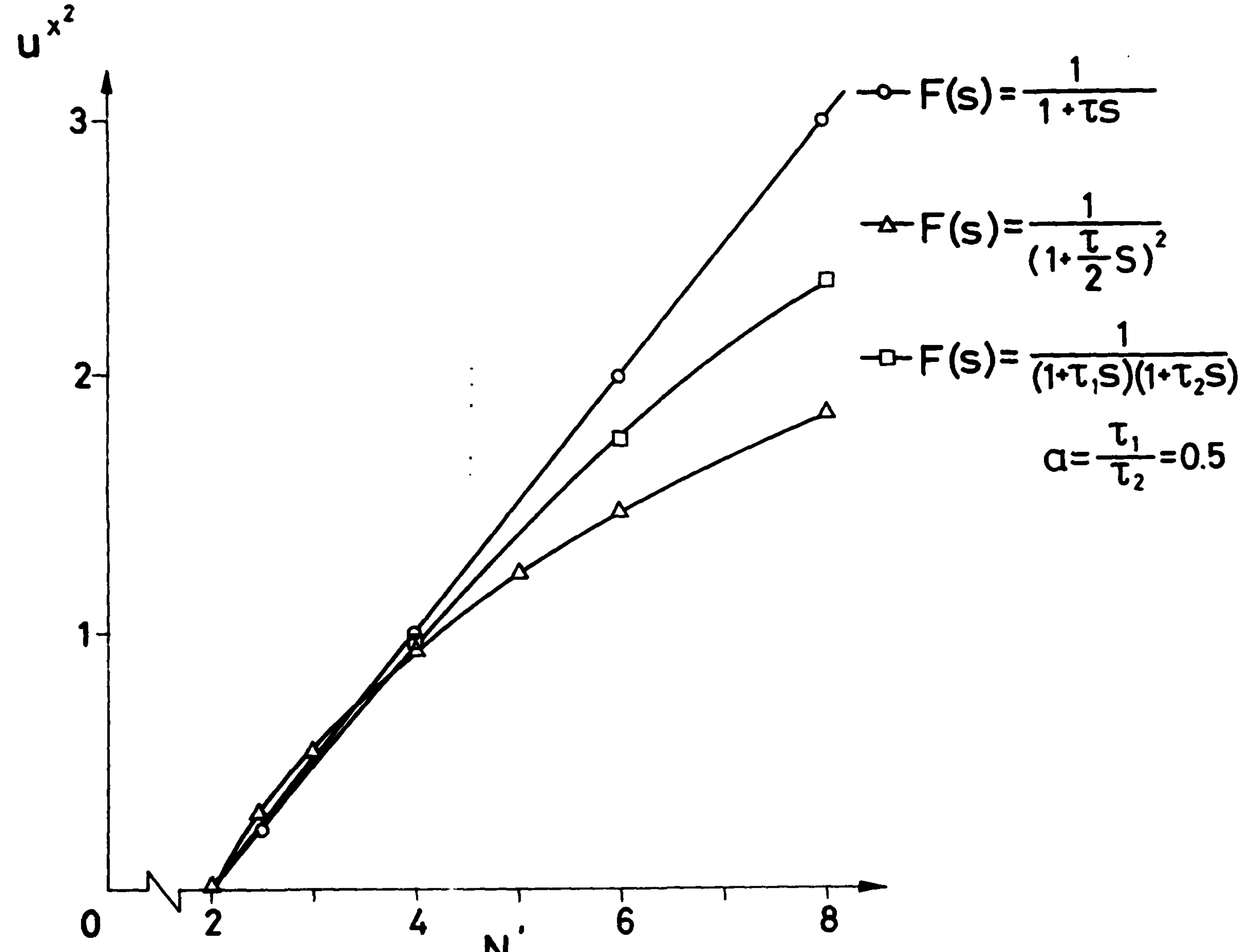

FIG. 8. Effect of missile transfer function on the optimal maneuver frequency.

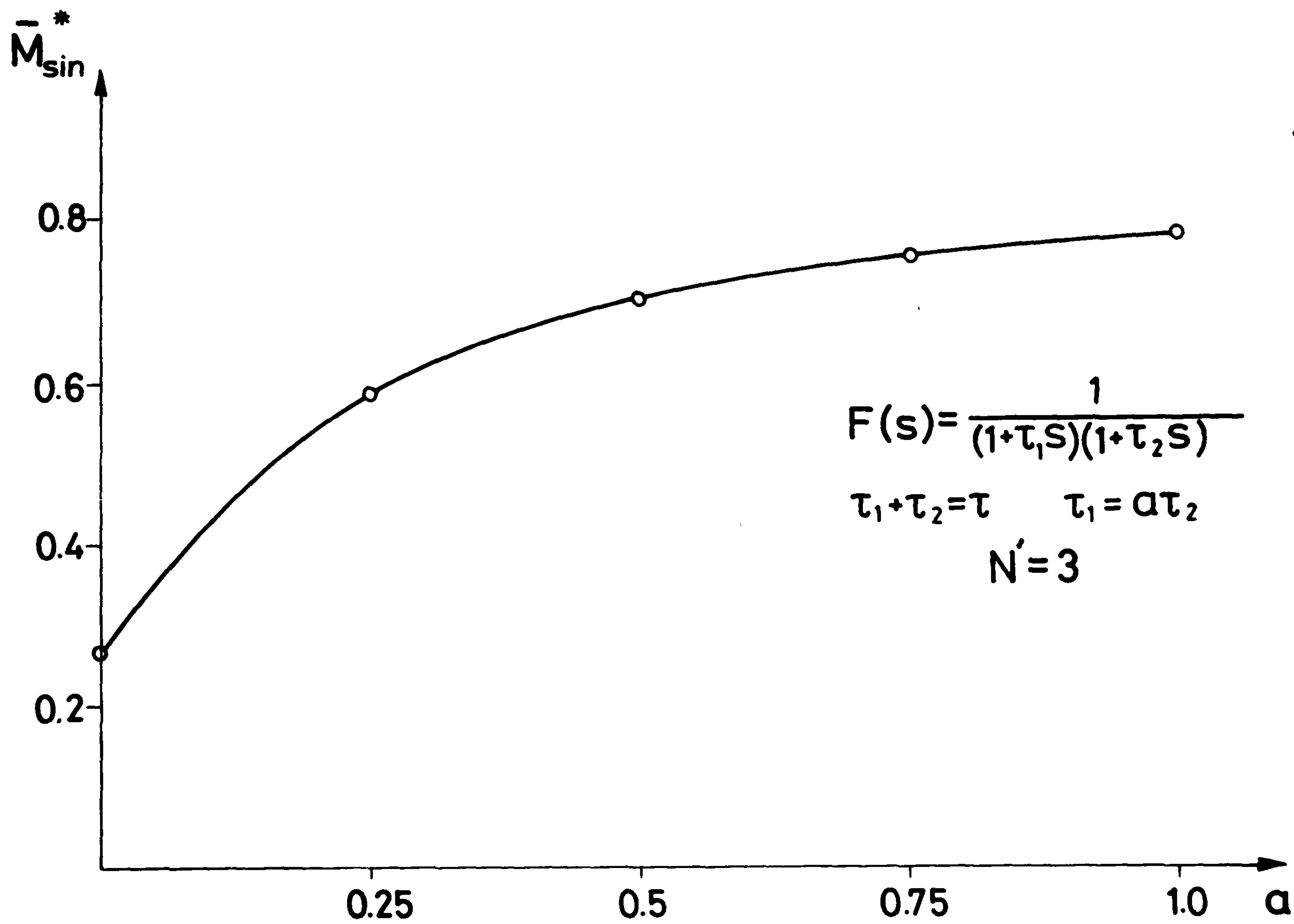

FIG. 9. Effect of the pole ratio in a second order transfer function on the RMS miss distance.

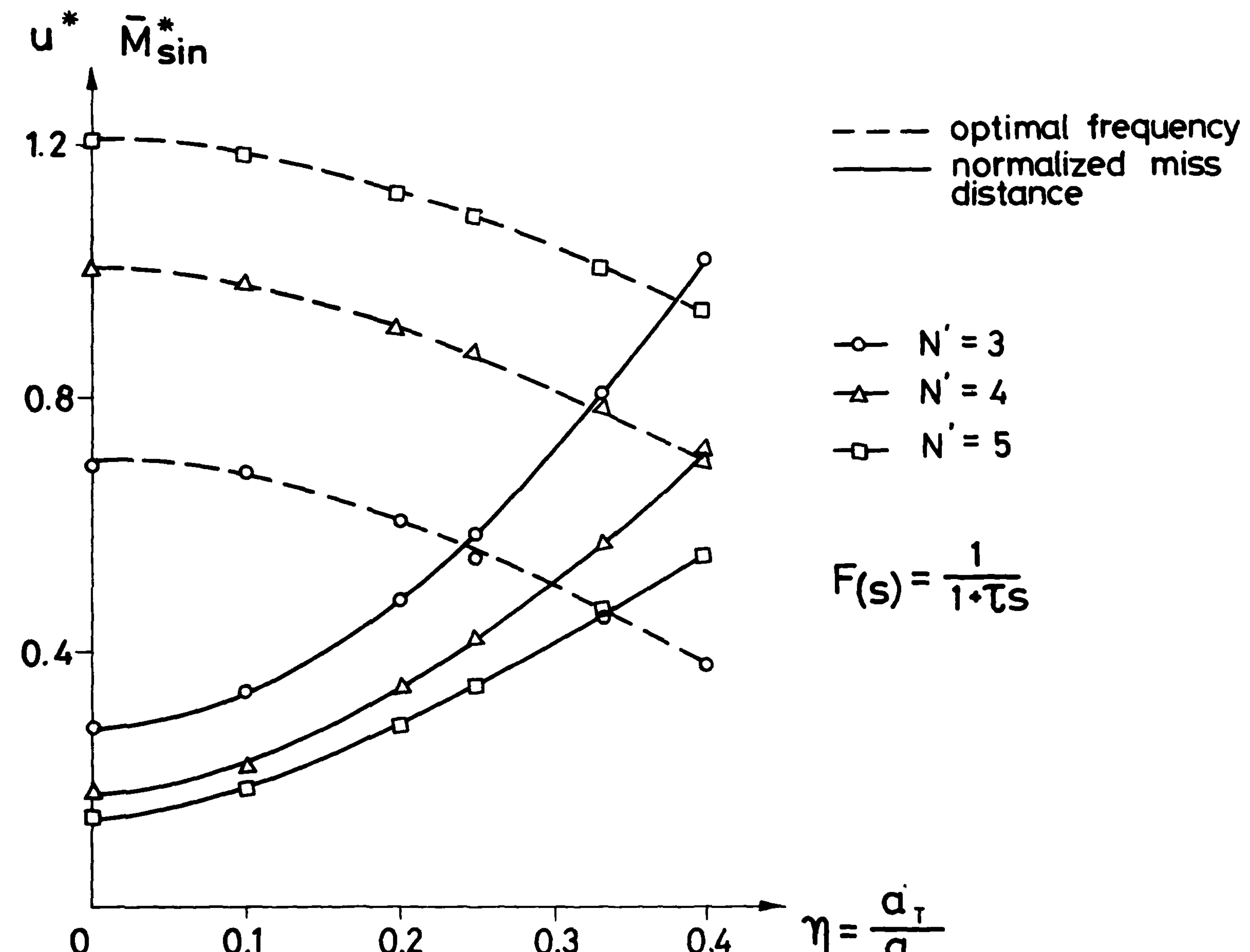

FIG. 10. Effect of the maneuver ratio on the optimal maneuver frequency and the RMS miss distance (limited acceleration output).

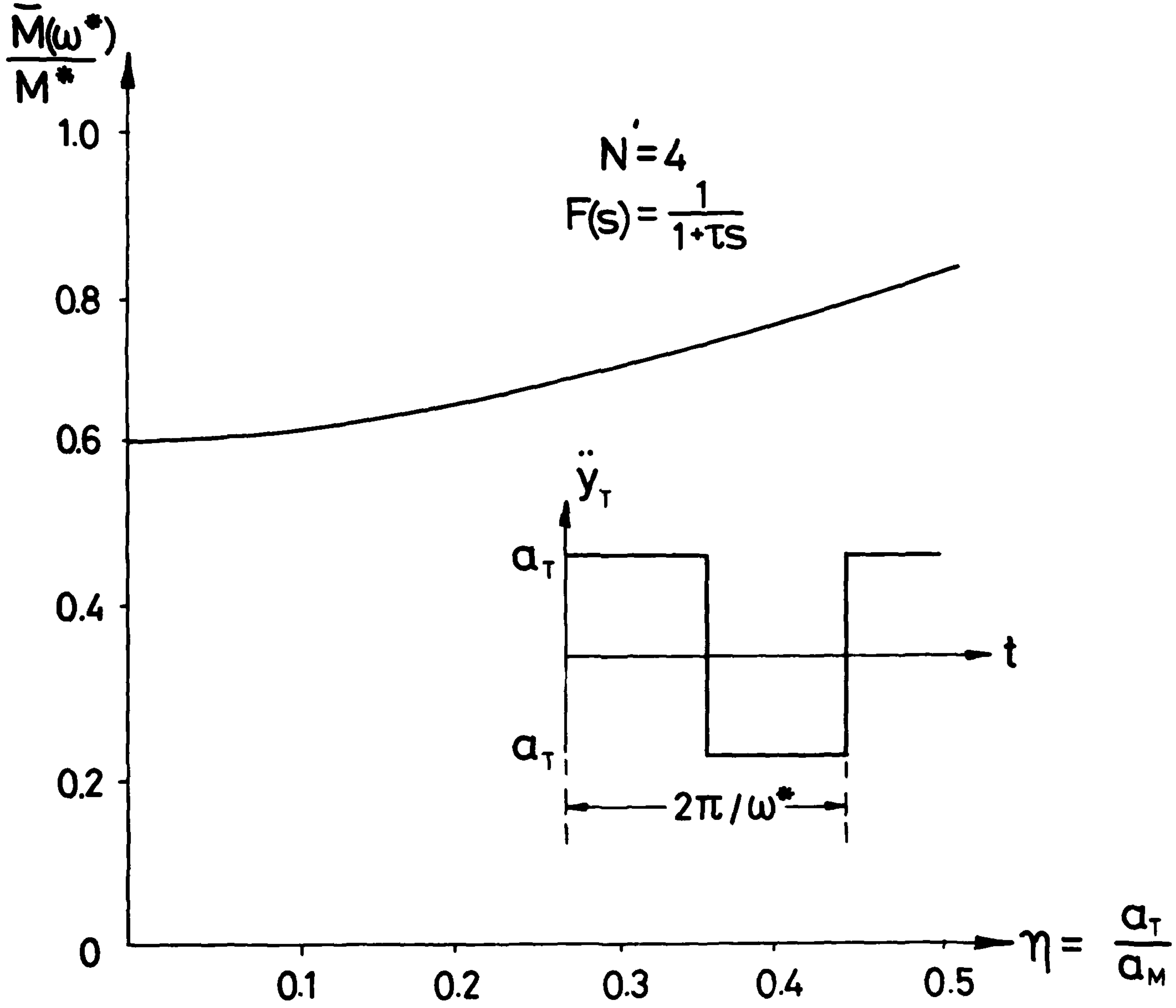

FIG. 11. Comparison of the RMS miss distance due to an *optimal periodical* maneuver to results of deterministic missile avoidance.